Strong Coupling Model for String Breaking on the Lattice.

I T Drummond, Department of Applied Mathematics and Theoretical Physics,
University of Cambridge, Silver Street, Cambridge, England CB3 9EW

A model for $SU(n)$ string breaking on the lattice is formulated using strong coupling ideas. It gives an explicit picture of string breaking, in the presence of dynamical quarks, as a mixing process between a string state and a two-meson state. An analysis of the Wilson loop shows that the evolution of the mixing angle as a function of separation may obscure the expected crossover effect. If a sufficiently extensive mixing region exists then an appropriate combination of transition amplitudes may help to reveal the effect. The sensitivity of the mixing region to the values of the meson energy and the dynamical quark mass is explored.

1. Introduction

Much effort is being devoted to dynamical quarks [1–6]. For a review see S Güsken [7]. For currently feasible values of the quark mass it is not expected that there will be dramatic effects on the computed hadron spectrum. String breaking is an effect that is *only* possible in the presence of dynamical quarks. Here we show explicitly that string breaking does occur on the lattice and shows up as a mixing phenomenon.

The physical situation envisaged in lattice calculations comprises a static quark and anti-quark separated by a spatial distance of R lattice units. The two static particles may either support a gauge string that runs between them or separately bind a quark and an anti-quark to create a two-meson state. When the string is stretched sufficiently, its energy coincides with the energy of the two meson state. In the neighbourhood of this critical separation the two different physical states mix permitting the occurence of transitions between them.

2. Strong Coupling Model

We recall the rules for evaluating simple graphs in the strong coupling limit of $SU(n)$ gauge theory [8,9].

1. A factor of $\left(\beta/2n^2\right)$ for each plaquette.
2. A factor of $2\kappa\left(1+\gamma.e\right)/2$ for each Wilson quark line in the direction of the unit vector e . Here κ is the standard quark hopping parameter.
3. Factors of (-1) and $1/n$ and a trace over the spin matrix factors for each internal quark loop.

2.1. Simple String Model

In the absence of dynamical quarks the correlation function of a string of length R over an imaginary time interval T is the standard $R\times T$ Wilson loop. In leading strong coupling approximation the above rules give for this string-string propagator

$$\mathcal{G}_{SS}(T)=\left[e^{-\sigma R}\right]^T \quad , \tag{1}$$

where the dimensionless string tension $\sigma = -\log\left(\beta/2n^2\right)$. The energy of the string state is $V(R)=\sigma R$.

2.2. Model for Mesons

Our model for mesons is one in which a light quark propagates along a static (anti-)quark line with a hopping parameter κ' that encodes the energy of the static bound state. Using this rule, we find that the meson propagator has the structure $g(T)=(1+\gamma_0)/2\,(2\kappa')^T$. In other parts of the diagrams of the model the hopping parameter retains the value κ related to the light quark mass.

The propagator for two mesons moving independently, each bound to its static quark, is

$$g^{(1)}(T)\otimes g^{(2)}(T)$$
$$=\left(\frac{1+\gamma_0}{2}\right)^{(1)}\otimes\left(\frac{1-\gamma_0}{2}\right)^{(2)}\left((2\kappa')^2\right)^T \quad . \tag{2}$$

In fact only a particular combination of quark-anti-quark spin wave functions is relevant to the mixing phenomenon. It is obtained by completing the light quark loop with the matrices $(1\pm\gamma_1)/2$, and including a factor of (-1) . The two-meson propagator becomes

$$\mathcal{G}_{MM}(T) = \frac{1}{2}\left((2\kappa')^2\right)^T \quad . \tag{3}$$

We have then $(2\kappa')^2 = e^{-E_M}$, where E_M is the combined energy of the two bound mesons.

3. String Breaking

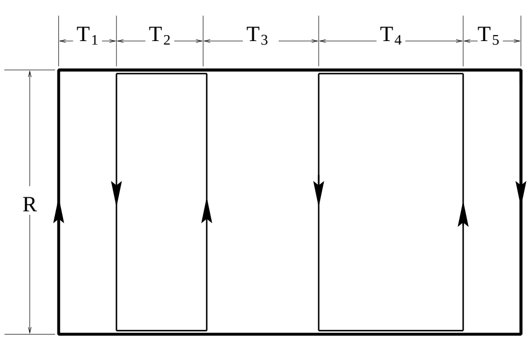

Figure 1. Wilson loop (heavy line) containing internal quark loops (light lines).

Our model for string breaking involves summing over planar graphs that incorporate transitions between the string and meson states. A typical graph is shown in Fig 1. The corresponding contribution to the Green's function is

$$\left[e^{-\sigma R}\right]^{T_1}\left[(2\kappa')^2\right]^{T_2}\left[e^{-\sigma R}\right]^{T_3}\left[(2\kappa')^2\right]^{T_4}\left[e^{-\sigma R}\right]^{T_1}$$

$$\left(-\frac{t}{n}\right)\left[(2\kappa)^2\right]^R[\sqrt{W}]^4\left(-\frac{t}{n}\right)\left[(2\kappa)^2\right]^R[\sqrt{W}]^4 \ ,$$

where

$$t = \mathrm{Tr}\left(\frac{1+\gamma_0}{2}\right)\left(\frac{1+\gamma_1}{2}\right)\left(\frac{1-\gamma_0}{2}\right)\left(\frac{1-\gamma_1}{2}\right) \ .$$

The factors involving κ (as opposed to κ') are associated with the *vertical* sides of the quark loops and the factor $\sqrt{W}$, where W is an energy, measures the rate of transition from a meson state to $q\bar{Q}$ pair. (This factor was omitted in the original formulation [10].) Note that the trace of the internal quark loop, which has the value $t = -\frac{1}{2}$, is equivalent to the quark-anti-quark spin projection mentioned previously.

The structure of our diagrams is as follows:

1. A factor $a = e^{-\sigma R}$ that propagates the string by one time step.
2. A factor $b = (2\kappa')^2$ that propagates the two-meson state by one time step.
3. A factor $c = (W/\sqrt{2n})\,(2\kappa)^R$ associated with the transition from string to two-meson state and vice-versa.

To describe the transition from an initial time to time T we need a 2×2 matrix of transition amplitudes

$$G(T) = \begin{pmatrix} G_{SS}(T) & G_{SM}(T) \\ G_{MS}(T) & G_{MM}(T) \end{pmatrix} \ . \tag{4}$$

then the above stepping procedure can be represented by $G(T+1) = AG(T)$, where the matrix A is given by

$$A = \begin{pmatrix} a & ac \\ bc & b \end{pmatrix} \ . \tag{5}$$

If for definiteness we set $G(0) = 1$, then $G(T) = (A)^T$. We can set $A = DO\Lambda O^{-1}D^{-1}$ where

$$D = \begin{pmatrix} \sqrt{a} & 0 \\ 0 & \sqrt{b} \end{pmatrix}, O = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \tag{6}$$

$$\Lambda = \begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix} \ . \tag{7}$$

The entries in Λ are the eigenvalues of A

$$\lambda_\pm = \frac{1}{2}\left\{(a+b) \pm \sqrt{(a-b)^2 + 4abc^2}\right\} \ . \tag{8}$$

The corresponding eigenvectors are the columns of DO. The mixing angle θ is

$$\tan\theta = \frac{-(a-b) + \sqrt{(a-b)^2 + 4abc^2}}{2\sqrt{ab}\ c} \ . \tag{9}$$

4. Transition Amplitudes

We assume that each transition amplitude begins and ends with an appropriate propagator. We find for the Wilson loop

$$\mathcal{G}_{SS}(T) = (1,0)A^{T-1}\begin{pmatrix} a \\ 0 \end{pmatrix} \tag{10}$$

with the result

$$\mathcal{G}_{SS}(T) = a\left(\cos^2\theta\ \lambda_+^{T-1} + \sin^2\theta\ \lambda_-^{T-1}\right) \quad . \tag{11}$$

Similar reasoning yields

$$\mathcal{G}_{MM}(T) = b\left(\sin^2\theta\ \lambda_+^{T-1} + \cos^2\theta\ \lambda_-^{T-1}\right) \quad , \tag{12}$$

$$\mathcal{G}_{MS}(T) = \sqrt{ab}\sin\theta\cos\theta\left(\lambda_+^{T-1} - \lambda_-^{T-1}\right) \quad . \tag{13}$$

All amplitudes see both exponentials, $e^{-E_\pm T} = \lambda_\pm^T$. The the lower energy exponential should dominate the asymptotic behaviour. However what is observed will be influenced by the behaviour of the mixing angle. When $\sigma R << E_M$ then $a >> b$ and $\theta \simeq 0$,$E_+ \simeq \sigma R$,$E_- \simeq E_M$. Under these circumstances the coupling to the state with energy E_- will vanish and only the exponential associated with E_+ will be observed as expected. When $\sigma R >> E_M$ then $b >> a$ and $\theta \simeq \frac{\pi}{2}$,$E_+ \simeq E_M$, $E_- \simeq \sigma R$. Now the coupling to the state with energy E_+ will vanish and only the exponential associated with E_- will be observed. That is, both above and below the crossover point, only the *original* string behaviour $e^{-V(R)T}$, will be observed. The movement of the mixing angle therefore obscures the crossover phenomenon in which the string energy is expected to be bounded by the energy of the two-meson state. Complementary results hold for $\mathcal{G}_{MM}(T)$. It is clear that $\mathcal{G}_{SM}(T)$ is suppressed *outside* the mixing region, where $\sin\theta\cos\theta \simeq 0$.

5. Mixing Region

To see mixing effects directly in the Wilson loop the mixing region in R, for which both $\sin\theta$ and $\cos\theta$ are reasonably large, must be of sufficient size to be resolved on the lattice. The range of the mixing region in R can be estimated as

$$\Delta R = \left(\frac{\pi}{2}\frac{dR}{d\theta}\right)_{R=R_c} = \pi\sqrt{\frac{2}{3}}\frac{W}{\sigma}e^{-m_q E_M/\sigma} \quad . \tag{14}$$

If we estimate the parameters in the model as $a^{-1} = 1.5$ GeV, $\sigma^{1/2} = 0.427$ GeV, $W \simeq E_M = 1.0$ GeV then the critical distance is $R_c = 8.22$ lattice spacings. For m_q=.1 GeV we then find ΔR=18.3 lattice spacings, while for m_q=.5 GeV we obtain ΔR=2.0 lattice spacings. We see then that if the dynamical quark is light enough and the meson energy not too high there is a reasonable chance of observing the string breaking within the mixing region. The exponential factor in eq(14) guarantees that if the meson energy is larger than 1 GeV the mixing region will be rapidly restricted as the value of m_q is raised. An appropriate combination of transition amplitudes may also help in identifying the mixing process. For example in the model the combination $b\mathcal{G}_{SS}(T) + a\mathcal{G}_{MM}(T)$ removes the influence of the mixing angle variation.

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